

ABSTRACT

The constraints that satellite data and other observations place on the forecasting of volcanic ash clouds has a precise and useful description in terms of the uncertainties in wind, ash source, and the position of ash clouds in the atmosphere. Using a Bayesian analysis, we define the uncertainty in the position of the ash cloud both in terms of the errors in satellite retrievals and the errors in projecting ash downwind from its source in the atmosphere. The constraints that satellite data place on transport models and on ash cloud forecasts is maximized using variational calculus and the Hessian of the misfit between satellite observations and model predictions, and this process can be rapidly automated. For any linear transport model, (i.e. NAME, FLEXPART, or Ash3d), the procedure here is exact, and quantitatively determines the quality of a forecast for any given set of satellite data, any given wind field, and any given model (or combination of models), with variable source parameters.

Here we illustrate the efficacy of our method using two different eruptions measured with two different satellite platforms; Eyjafjallajokull in 2010 measured with SEVIRI, and Kasatochi in 2008 measured with MODIS. These examples illustrate conditions which illustrate the evolving constraints on forecasts of volcanic ash clouds during the same eruptive sequence.

INTRODUCTION

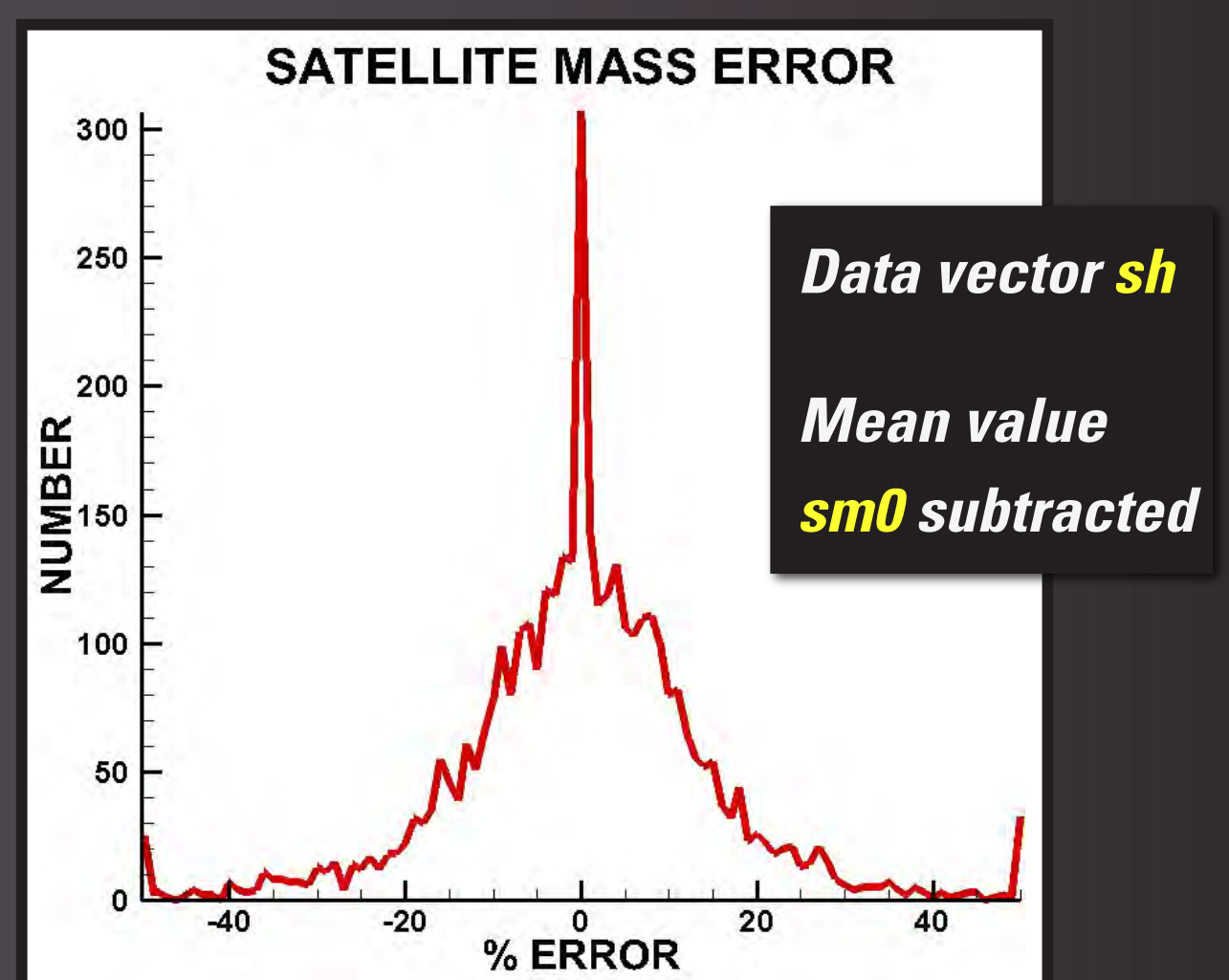
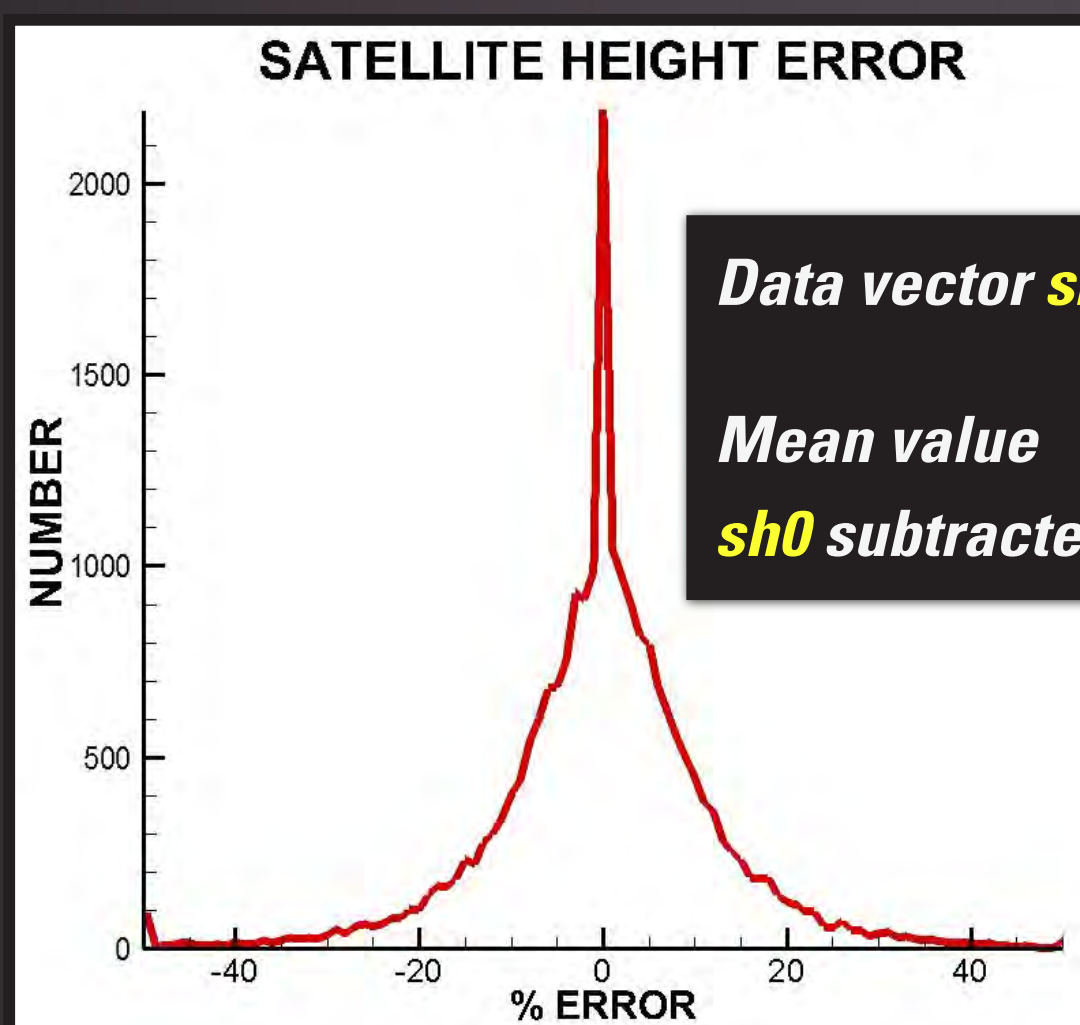
At the IUGG-WMO Workshop on Ash Dispersal Forecasts (WMO) meeting in Geneva on volcanic ash clouds, uncertainty in cloud forecasts was highlighted as one of the primary concerns in hazard mitigation for these clouds. A comparison of models for the same eruption parameters at a previous WMO meeting in 2010 illustrated how diverse the models used by VAACs are when compared in detail for deposition. A similar study was recently done comparing model ash clouds with real ash clouds, with similar results.

These comparisons show that errors introduced by models, by data used by models, and by errors in satellite data used to constrain models result in variable outcomes, compromising ash cloud forecasts. We address this problem of uncertain forecasts from uncertain data quantitatively.

OUR ANALYSIS ANSWERS TWO SPECIFIC QUESTIONS:

- 1 How does uncertainty affect forecasts of volcanic ash clouds?
- 2 How do we maximize the constraints observations place on model results and forecasts?

DATA ERRORS & MODEL MISFITS



The error in the **prior** satellite data D is described with the prior distributions above, which combined produce variance $1/\alpha$

$$p(\boldsymbol{\theta} | \alpha) = N(\boldsymbol{\theta} | \mathbf{0}, \alpha^{-1}\mathbf{I})$$

Errors in transport models and in winds \mathbf{w} are encapsulated into an error parameter β through the **likelihood** of model M inputs

$$p(D(t^n) | \boldsymbol{\theta}, \beta) = \exp \left[-\frac{\beta}{2} \sum_{j=1}^{j_{\max}} \sum_{i=1}^{i_{\max}} \left(M_{ij}(\boldsymbol{\theta}, \mathbf{w}(t^n)) - sm_{0_{ij}} \right)^2 + \left(M_{ij}(\boldsymbol{\theta}, \mathbf{w}(t^n)) - sh_{0_{ij}} \right)^2 \right]$$

BACKGROUND

From Bayes theorem, the posterior distribution over the model parameters is proportional to the product of both the **prior** and **likelihood** distributions:

$$p(\boldsymbol{\theta} | D, \alpha, \beta) \propto p(D | \boldsymbol{\theta}, \beta) p(\boldsymbol{\theta} | \alpha)$$

For forecasting this posterior must be normalized by the integral over all ranges of input parameters $\boldsymbol{\theta}$, which determines the **evidence** $p(D | \alpha, \beta)$ given by the data constraints:

$$p(D | \alpha, \beta) = \int p(D | \boldsymbol{\theta}, \beta) p(\boldsymbol{\theta} | \alpha) d\boldsymbol{\theta}$$

METHOD

The evidence contains the information needed to quantitatively resolve uncertainty in forecasting ash clouds

The second derivative \mathbf{A} of the log posterior is scaled by the errors and can be written

$$\mathbf{A} = -\nabla\nabla \ln p(\boldsymbol{\theta}_{MAP} | D(t^n), \alpha, \beta) = \alpha\mathbf{I} + \beta\mathbf{H}$$

where \mathbf{I} is the identity matrix and \mathbf{H} is the Hessian of the misfit of model and satellite data.

The partial derivatives of the Hessian \mathbf{H} with respect to β or α shows, *in the limit when the number of satellite observations greatly exceed the number of model parameters*, that maximum data constraints occur with

$$\frac{1}{\alpha} = \frac{\mathbf{sm}^T \mathbf{sm} + \mathbf{sh}^T \mathbf{sh}}{\# \text{ parameters}} \quad \frac{1}{\beta} = \frac{\sum_N (M(\boldsymbol{\theta}, t) - \mathbf{D}(t))^2}{\# \text{ satellite observations}}$$

Eyjafjallajokull
volcano

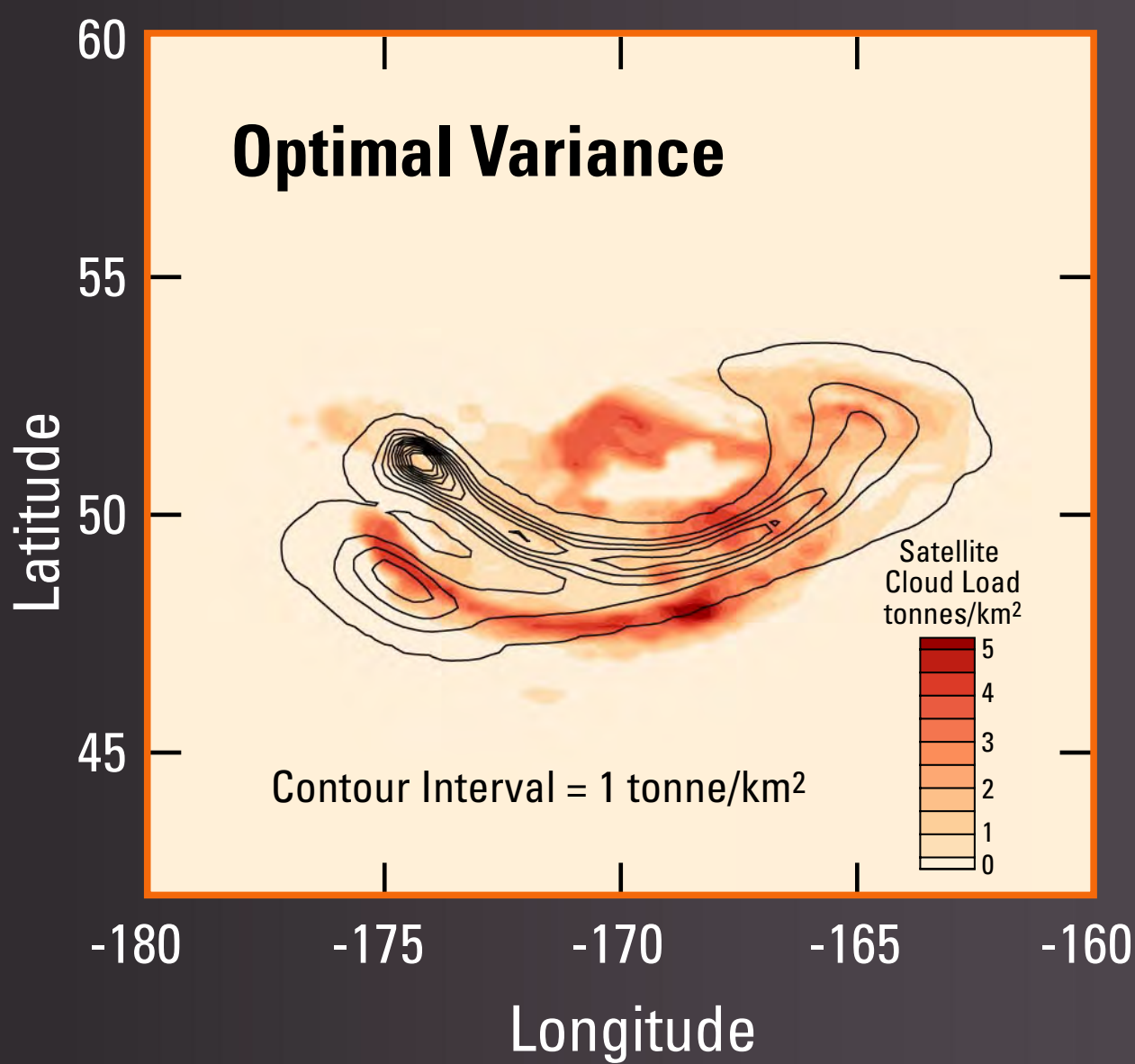
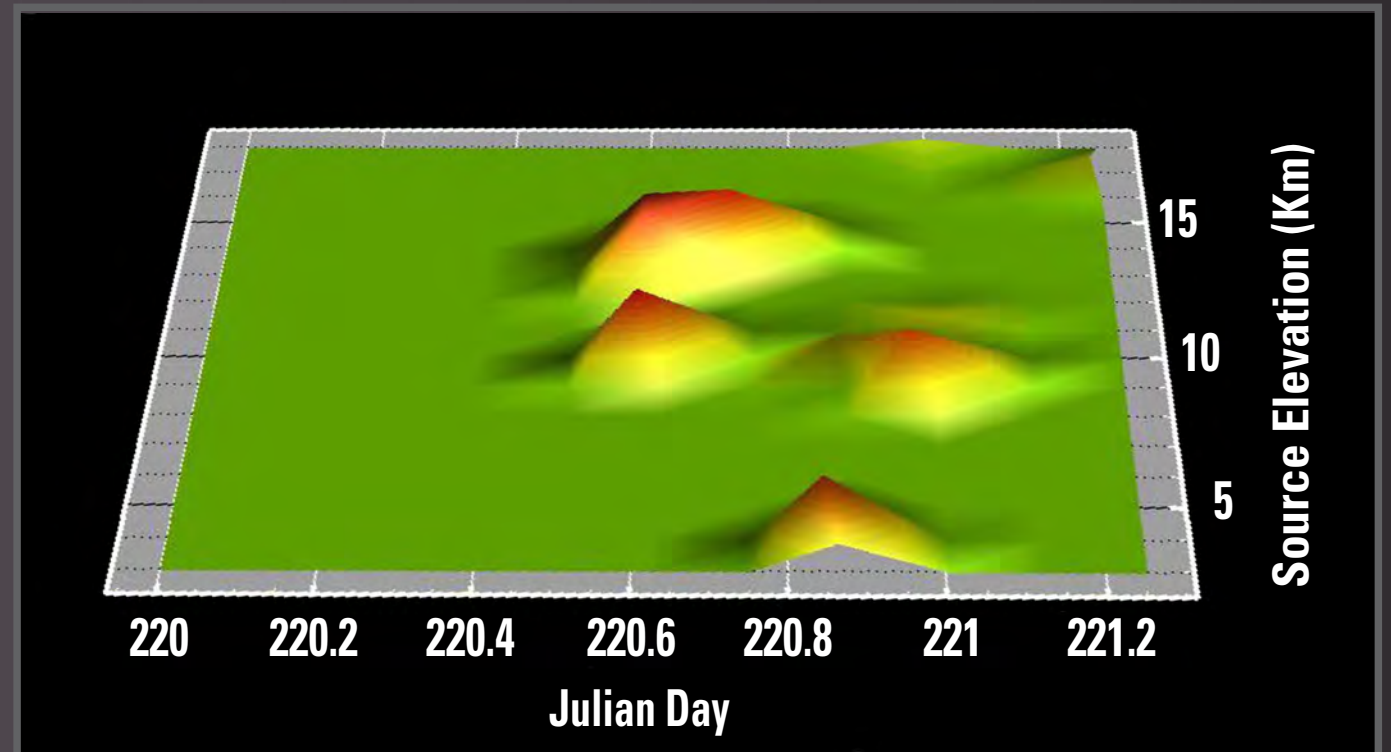


Eyjafjallajokull volcano, Iceland, erupting in May, 2010. The ash cloud downwind of the volcano occupies a narrow range of altitudes, and this behavior produces a strong, dominant posterior distribution in source parameters.

EXAMPLE 1: Kasatochi Eruption, 2008

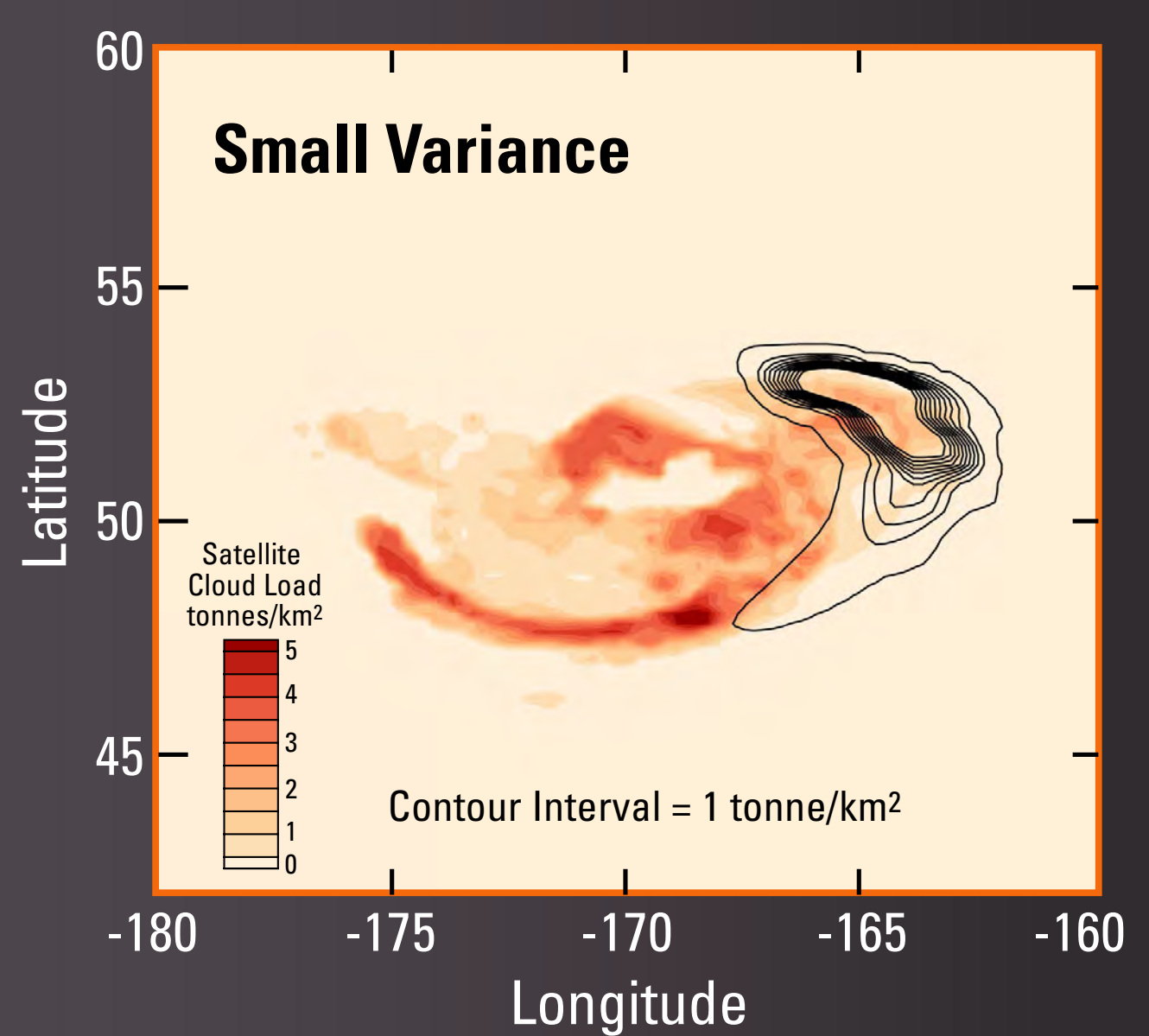
11 HOUR FORECAST

- Log Posterior of the source of the ash cloud, from MODIS data, with each mass eruption rate.
- The shape of these peaks determines the uncertainty.



Optimal Variance

Use of optimal errors (full approximation of peaks above) results in maximum data constraints.



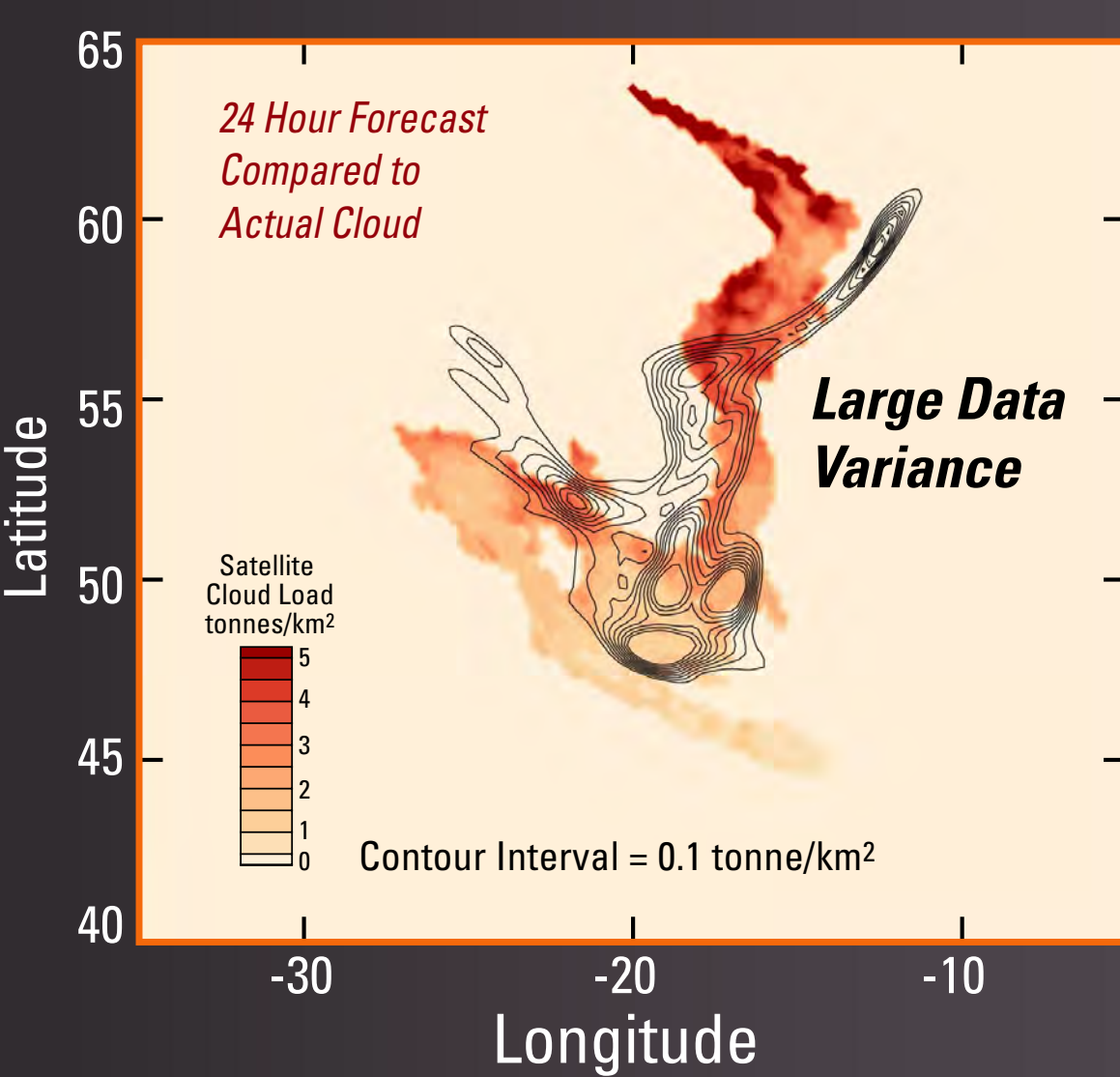
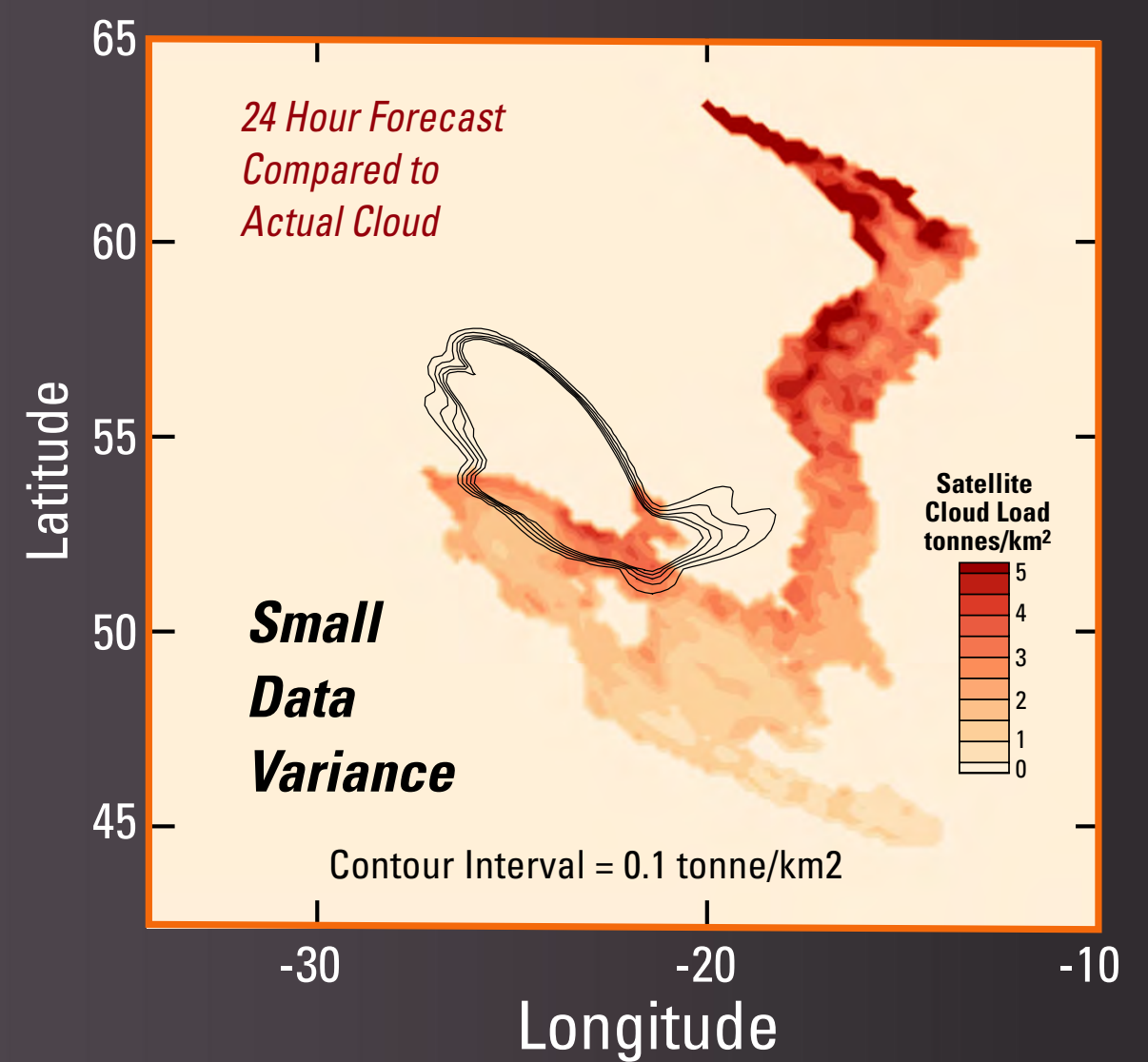
Small Variance
Underestimating errors (small variance, or narrow approximation of peaks above) eliminates good solutions.

EXAMPLE 2: Eyjafjallajökull Eruption, 2010

24 HOUR FORECAST

Small Data Variance

A variance that is underestimated wastes data, as satellite constraints that would be applied are minimized.

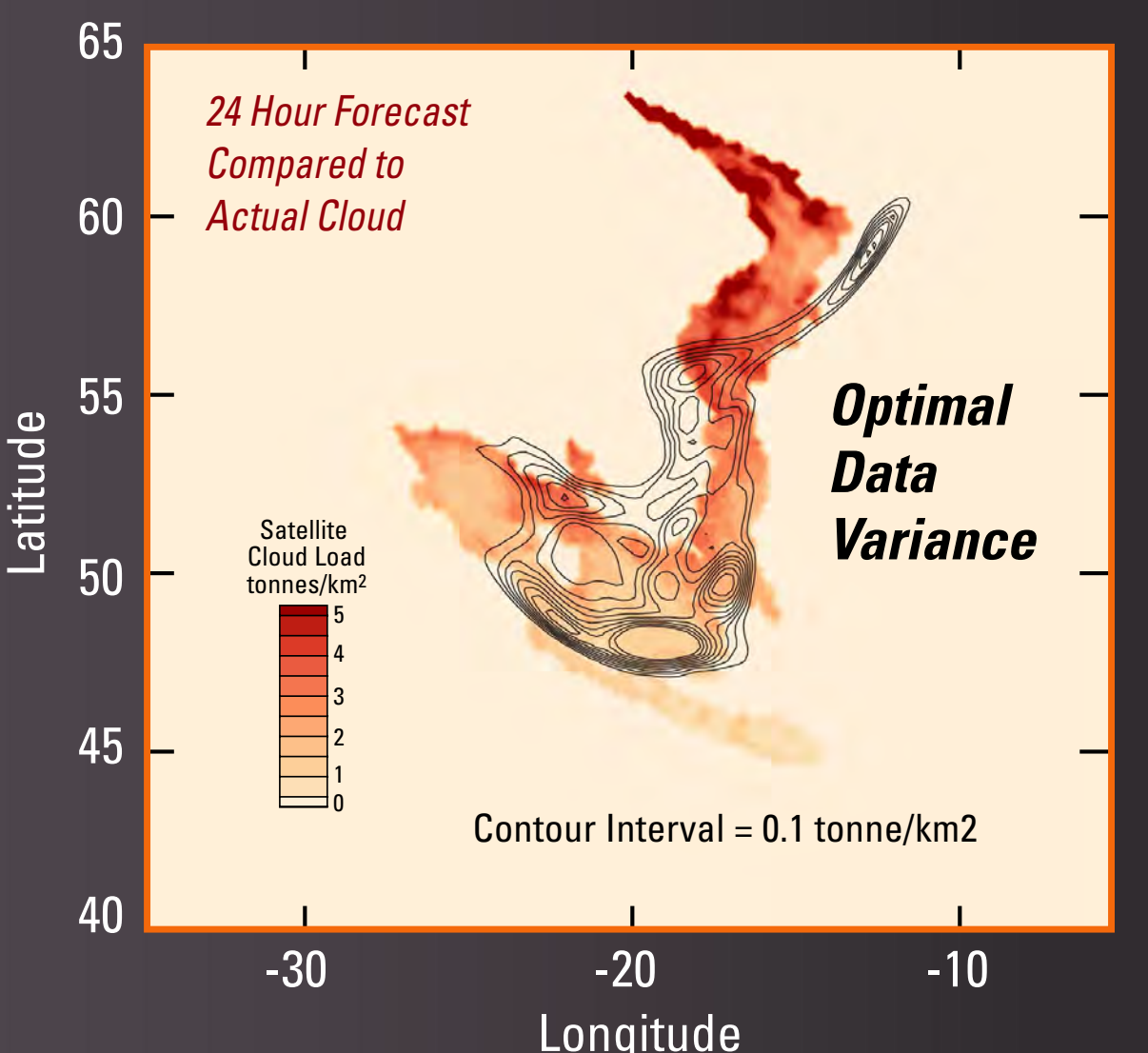


Large Data Variance

If the estimate of errors is too liberal, then poor model solutions contaminate the forecast.

Optimal Data Variance

Use of the errors inherited from the satellite data and measured from the misfits of model comparisons to satellite data, as shown, provide the optimal forecast.



CONCLUSIONS

- 1 The error inherited from these satellite data directly affect forecasts.
- 2 The uncertainty in forecasting is a combination of these inherited errors, errors in transport models, and errors in wind.
- 3 A Bayesian framework for making forecasts provides the means to measure misfits of models with data, and uses this misfit to correctly determine the optimal data constraints to make a forecast.